Fracture Mechanics, Damage and Fatigue

Chapter 1 - Introduction

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Introduction

• Fracture phenomenon
  - Fracture is a problem that society has faced for as long as there have been man-made structures
  - The problem may actually be worse today than in previous centuries, because more can go wrong in our complex technology society
  - Economic studies estimated the cost of the fracture in industrialized countries, about 4% of the gross national product

• Failures generally falls into one of the following categories
  - Negligence during design or construction of the structure
  - Application of new design or new material, which can produces an unexpected (and undesirable) result (type 2 failures)

• Brittle fracture of the World War II Liberty ships is an example of type 2 failures
  - New design: the ships were the first to have an all-welded hull
  - These ships could be fabricated must faster and cheaper than earlier riveted designs

- Development and propagation of cracks in welded joints
Another example of type 2 failures: use of polymers
- Provide a number of advantages over metals
- Polyethylene (PE) is currently used in natural gas transportation systems
- One advantage of PE piping is that maintenance can be performed on a small branch without shutting down the entire system: a local area is shut down by applying a clamping tool to the PE pipe and stopping the flow of gas
  ⇨ Crack development in the pinched parts

Crack growth in a PE pipe as a result of pinch clamping
• **Disasters fatigue failure (1)**
  - Meudon railway accident May 8, 1942
  (First disaster of railway history)

The accident was caused by the failure of one of the axles of the damaged locomotive.

William Rankine (1820-1872), examining the fracture facies of the broken axles during the accident, showed that it was a fatigue rupture.

• **Disasters due to resonance failure (2)**
  - Breaking the bridge of Basse-Chaîne à Angers (1850)
  (failure due to resonance)
- **Disasters due to resonance failure (3)**
  - Failure of Takoma Narrow Bridge San Francisco 1940

The steady wind of 42 miles per hour was sufficient to generate and maintain the vibrations of the bridge to the resonant frequency. After an hour of torsional vibrations, the bridge finally collapsed.

- **Disasters fatigue failure (4)**
  - Accident DC 10 - United Airlines Flight 232 on 19-7-89

Failure due to fatigue cracking in the metal of one of the turbine blades and not detected in the last inspection. The origin of this crack comes from a manufacturing defect of the alloy composing the turbine blade.
• **Disasters fatigue failure (5)**

Derailment of the German ICE in Eschede in June 1998 caused by the rupture of a cracked wheel

The accident was caused by fatigue crack propagation in a low noise wheel that had just been put into service.

• **History of the failure**

- History shows that man has always tried to avoid failure
- The old structures were stressed in compression (pyramids, Roman bridges ...)
  - Stone, brick, mortar ... are fragile tensile materials
- Before the industrial revolution, the loadings were of compression
- After the industrial revolution, the loadings were in tension with the use of steel
- The fatigue failures occur at stresses below the yield stress $\sigma < \sigma_y$.
- We tried to solve these problems by oversizing, but we ran into the problem of weight.
  $\Rightarrow$ Which led to the development of Fracture Mechanics.
- The first break tests were performed by Leonardo da Vinci in the 15th century.

- Leonardo da Vinci discovered that the tensile strength varies inversely with the length of the wire tested.
- The defects control the resistance of the wire $\Rightarrow$ when the wire is longer, the probability of failure is higher.

• **Failure Griffith theory**
  - Griffith establishes a direct relationship between the defect size and the failure stress $\sigma_R$.
  - Griffith assumes that the failure occurs when the propagation energy reaches the specific energy of the material, denoted $\gamma_S$.
    - This theory is valid only for brittle solids.
  - In ductile materials, where plastic flow occurs, the plastic work per unit area of surface created, denoted $\gamma_P$, is typically much larger than $\gamma_S$.
  - In 1948, Irwin suggested a modification of this theory by taken into account the plastic work in the energy balance.
• **Failure Griffith theory**

- In 1956, Irwin developed the concept of energy release rate, denoted \( G \)
- In 1957, Irwin developed the concept of Stress Intensity Factor \( K \) (based on the work of Westergaard and Mushkhélishvili) to describe the fields of stress and strain at the tip of a crack
- The SIF \( K \) and the energy release rate \( G \) are two concepts of linear elastic fracture mechanics interlinked; a simple relationship exists between these two parameters
- These two concepts have since been widely used to describe the propagation of cracks
- The endurance curves have been supplemented by the crack propagation curves

- There was an intensification of research between 1960 and 1980 with the confrontation of two schools
  - Supporters of the LEFM approach using the SIF \( K \) (with plastic zone correction)
  - Those interested by the crack tip opening displacement (CTOD, \( J \))
- Since the 1980s, research has focused on:
  - viscoplastic behavior (high temperature ductile materials, creep, fatigue-creep)
  - viscoelastic behavior (polymeric materials)
  - the behavior of composites (delamination, impact effects ...)

- New, more recent approaches attempt to link the local microscopic behavior at global macroscopic behavior (micro-macro models)

**Use of Fracture Mechanics in the design of structures**

- Classic two-parameters approach
  - Sizing the structure so that the applied stress remains below the elastic limit \( \sigma_a < \sigma_E \)

- LEFM approach with three parameters
  - Sizing the structure so that the SIF \( K \) remains below the toughness of material \( K < K_C \) (or \( G < G_C \))
• **Energy criterion**

*Griffith energy criterion for brittle materials*

*Irwin - Orowan energy criterion for ductiles materials*

- The crack propagation occurs if the energy provided is sufficient to overcome the resistance of the material
  \( (\gamma_S, \gamma_P, ...) \)

- Griffith energy \( G \) is defined by the variation of energy, per unit of cracked surface, associated with the propagation of a crack in a linear elastic material

- According to this criterion, the failure occurs when \( G \) reaches a critical value \( G_C \)

- \( G_C \) is a measure of the toughness of the material, i.e., its ability to resist the propagation of a crack-type defect

• **Energy criterion (continued)**

- It is assumed that the principle of similarity is verified, i.e., that \( G_C \) is independent of the geometry of the solid loaded

- To determine the energy \( G \), a plate with a small crack is considered

  (the plate is an infinite medium when considered across the crack)
Stress Intensity Factor (SIF) concept

This concept is characterized by SIF $K$ – a single parameter to describe $\frac{\sigma^*}{\mu}$

$$
K = \sigma^* \sqrt{a}
$$

$$
\begin{align*}
\sigma_{xx} &= \frac{K_i}{\sqrt{2\pi r}} \cos \theta \left( 1 - \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\
\sigma_{yy} &= \frac{K_i}{\sqrt{2\pi r}} \cos \theta \left( 1 + \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\
\tau_{xy} &= \frac{K_i}{\sqrt{2\pi r}} \frac{\theta}{2} \cos \frac{3\theta}{2}
\end{align*}
$$

$$
G = \frac{\pi(\sigma^*)^2 a}{E} = \frac{K_i^2}{E} \\
G_c = \frac{\pi\sigma_c^2 a}{E} = \frac{K_i^2}{E}
$$
**Concept of damage tolerance**

SIF $K$ is used to describe the crack propagation

![Stress vs. Time](image)

\[
\frac{da}{dN} = C(\Delta K)^m \quad \text{(Paris’s Law)}
\]

- **Concept of damage tolerance**
  - The structures are dimensioned taking into account the presence of cracks
  - Tolerating their propagation from an initial size to an acceptable size

![Diagram](image)

\[
N = \int_{a_0}^{a_c} \frac{da}{C(\Delta K)^m}
\]
Classification of the concepts of the fracture mechanics according to the nature of the materials to which they apply

- LEFM
  (Brittle materials, confined plasticity)
  • Hardening precipitation aluminium alloys
  • High yield strength steels (maraging steel …)
  • Monolithic or composite ceramics

- NLFM or EPFM
  (Ductile materials, high plasticity)
  • Low and average yield strenght steels \( \sigma_E \)
  • Austenic steels

- DFM
  (Materials loaded at high speed of deformation)

- VEFM
  (Polymer materials)

- VPFM
  (Metals and ceramics loaded at high temperature)

Objectives and consequences of fracture mechanics

- The determination of the stress field in the vicinity of a notch or a crack

- Determining the ability of a material to resist crack growth by means of internationally validated standardized tests

- The development of new methods of structural analysis, and reliable inspection and maintenance procedures and cost for optimal operation

- Prevention of life of structures having known dimensions defects
• **Fatigue damage**

Service structures are generally subject to cyclic mechanical and/or thermal stresses. These stresses, although lower than the elastic limit of the material, can lead to failure: it is the process of fatigue damage.

This damage has two stages. Initially, a microcrack is initiated near a stress concentration zone; this initiation is followed by a crack propagation at the microscopic scale, invisible to the naked eye. Secondly, the crack propagates at the macroscopic scale to failure.

The fatigue life is therefore naturally decomposed during the initiation period and the propagation period. For practical reasons, the crack growth on the microscopic scale, i.e., cracking over a length of some grains, is included in the initiation period.

Microscopic studies of the early 20th century\(^1\) have shown that fatigue damage in metallic materials begins with the initiation of microcracks in areas where deformation is localized. These zones, called slip bands, appear on the surface of the tested pieces after intrusions and extrusions have been formed according to the mechanism shown schematically in the figure below.

The slip bands leave the sample surface, traces obtained after a slight attack with chemical reagent, that can be observed in figure (a) below. This figure also shows the grain boundaries.

The micro crack initiation that can be observed in figure b (red arrow), is the result of a cyclic sliding mechanism in the slip band. The shear stresses that cause this cyclic shift are not uniformly distributed on a microscopic scale. In some grains on the surface of the material, the conditions are more favorable for the development of cyclic shift.

Chapter 2 – Complex formulation of the plane theory of elasticity

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Contents

- Plane theory of elasticity
- Airy stress function
- Complex representation of stresses and displacements
- Complex representation of resultant force and moment
Elasticity Equations

- Behavioural equations - Hooke’s Law
- Equilibrium equations
- Compatibility equations

Solutions satisfying the boundary conditions

Linear Elastic Materials (Hooke’s Law)

\[
\bar{\varepsilon} = \frac{1 + v}{E} \sigma - \frac{v}{E} (\text{trace} \bar{\sigma}) \bar{I} \quad \bar{\sigma} = 2\mu \bar{\varepsilon} + \lambda (\text{trace} \bar{\varepsilon}) \bar{I}
\]

\[
\begin{align*}
\mu &= \frac{E}{2(1 + v)} \\
\lambda &= \frac{E \nu}{(1 + v)(1 - 2v)} \\
\end{align*}
\]

\[
\begin{align*}
v &= \frac{\lambda}{2(\lambda + \mu)} \\
E &= \frac{3\lambda + 2\mu}{\lambda + \mu} \Rightarrow \frac{v}{E} &= \frac{\lambda}{2\mu(3\lambda + 2\mu)}
\end{align*}
\]

Plane elasticity

Plane stress

\[
\varepsilon_x = \frac{1}{2\mu} \left[ \sigma_x - \frac{\lambda'}{2(\lambda + \mu)} (\sigma_x + \sigma_y) \right] \\
\varepsilon_y = \frac{1}{2\mu} \left[ \sigma_y - \frac{\lambda'}{2(\lambda + \mu)} (\sigma_x + \sigma_y) \right] \\
\varepsilon_{xy} = \frac{1}{2\mu} \sigma_{xy}
\]

Plane strain

\[
\begin{align*}
\lambda' &= \lambda \\
\lambda' &= \frac{2\lambda \mu}{\lambda + 2\mu}
\end{align*}
\]

for plane strain

for plane stress

\[
\begin{align*}
\varepsilon_x &= \frac{1}{2} \left[ \sigma_x - \frac{\lambda'}{2} (\sigma_x + \sigma_y) \right] \\
\varepsilon_y &= \frac{1}{2} \left[ \sigma_y - \frac{\lambda'}{2} (\sigma_x + \sigma_y) \right] \\
\varepsilon_{xy} &= \frac{1}{2} \sigma_{xy}
\end{align*}
\]

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Resolution by the method of Airy stress function

- **Equilibrium equations**

\[ \bar{\text{div}} \sigma + \bar{f} = 0 \]

\[ \sigma_{y,j} + f_i = 0 \]

Body forces are derivable

\[ \bar{f} = -\text{grad} V \]

where \( V = V(x, y) \)

\[ \left\{ \begin{array}{l}
X = -\frac{\partial V}{\partial x} \\
Y = -\frac{\partial V}{\partial y}
\end{array} \right. \]

\[ \left\{ \begin{array}{l}
\sigma_{x,x} + \sigma_{y,y} + X = 0 \\
\sigma_{x,y} + \sigma_{y,x} + Y = 0
\end{array} \right. \Rightarrow \left\{ \begin{array}{l}
(\sigma_x - V)_{,x} + \sigma_{y,y} = 0 \\
\sigma_{x,y} + (\sigma_y - V)_{,y} = 0
\end{array} \right. \]

\[
\begin{align*}
\sigma_x - V &= A_{yy} \\
\sigma_y - V &= A_{xx} \\
\sigma_{xy} &= -A_{xy}
\end{align*}
\]

\[ \text{Airy stress function} \]

- **Compatibility equations**

\[ \epsilon_{ij,kl} + \epsilon_{kl,ij} - \epsilon_{il,jk} - \epsilon_{jk,il} = 0 \]

\((ijkl) = (1212), (1213) + \text{circular permutation on indices}\)

\[
\text{Plane Elasticity} \rightarrow \left\{ \begin{array}{l}
\epsilon_{x,y,y} + \epsilon_{y,x,x} = 2\epsilon_{x,y,xy} \\
\epsilon_{x,x,x} = \epsilon_{y,y,y} = \epsilon_{z,z,xy} = 0
\end{array} \right.
\]

\[
\begin{align*}
[(1-v)\sigma_x - v\sigma_y]_{yy} + [(1-v)\sigma_y - v\sigma_x]_{xx} &= 2\sigma_{xy,xy} \\
(1-v)(\Delta A + 2V) - (A_{xy} + V)_{,xx} - (A_{xx} + V)_{,yy} &= -2A_{xxyy}
\end{align*}
\]

\[
\Delta(\Delta A) + \frac{1-2v}{1-v} \Delta V = 0
\]

\[
\Delta(\Delta A) + \frac{2\mu}{\lambda + 2\mu} \Delta V = 0
\]
**Body forces = forces of the gravity**

\[ \mathbf{f} = \rho \mathbf{g} = -\rho \mathbf{g} y \]

\[ V(x, y) = V(y) = \rho g y + V_0 \]

\[ \Delta(\Delta A) + \frac{2\mu}{\lambda+2\mu} \Delta V = 0 \]

\[ \Delta(\Delta A) = 0 \]

**If the body force vanishes**

\[ \sigma_x = A_{yy} \]

\[ \sigma_y = A_{xx} \]

\[ \sigma_{xy} = -A_{xy} \]

\[ \text{avec } \Delta(\Delta A) = 0 \]

---

**Polar coordinate formulation**

Some problems of plane elasticity are more easily treated in polar coordinates.

\[ A(x, y) \rightarrow A(r, \theta) \]

\[ \Delta(\Delta A) = 0 \rightarrow \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \left( \frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A}{\partial \theta^2} \right) = 0 \]

\[ \sigma_r = \frac{1}{r} \frac{\partial A}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A}{\partial \theta^2} \]

\[ \sigma_\theta = \frac{1}{r} \frac{\partial^2 A}{\partial r \partial \theta} \]

\[ \tau_{r\theta} = -\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial A}{\partial \theta} \right) \]

\[ \varepsilon_r = \frac{\partial u_r}{\partial r} \]

\[ \varepsilon_\theta = \frac{u_\theta}{r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} \]

\[ \gamma_{r\theta} = \frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \]

---
Tutorial 1: Study of a gravity water dam

\[ \gamma_e \] is the specific weight of water  
\[ \gamma_b \] is the specific weight of the concrete

* Determining the stress field according to \( \gamma_e \), \( \gamma_b \) and \( \alpha \).

* For which values of \( \alpha \) the dam does not rise assuming:
  a- no infiltration (seepage) under the dam
  b- infiltration under the dam

Calculate the numerical values of \( \alpha \) taking \( \gamma_b = 2 \gamma_e \).

The body forces in the concrete, taken into account the choice of axes, are:

\[ \bar{f} = \gamma_b \bar{x} \]  
\[ \bar{f} = -\nabla V \Rightarrow V = -\gamma_b x \]

The wall \( OA \) of the dam is subjected to the hydrostatic pressure \( P \) of the water which varies linearly with the depth:

\[ \bar{P} = \gamma_e \bar{x} \bar{y} \]

The Airy stress function \( A \) is a polynomial of order 3, so that the derivative of \( A \) in the order 2 leads to a linear function:

\[ A(x, y) = ax^3 + bx^2y + cxy^2 + dy^3 \]

\[ \begin{align*}
\sigma_x - V &= A_{yy} \\
\sigma_y - V &= A_{xx} \\
\sigma_{xy} &= -A_{yx}
\end{align*} \]
The stress tensor components are given by:

\[
\begin{align*}
\sigma_x - V &= \frac{\partial^2 A}{\partial y^2} \Rightarrow \sigma_x = -\gamma_x + 2\alpha x + 6\alpha y, \\
\sigma_y - V &= \frac{\partial^2 A}{\partial x^2} \Rightarrow \sigma_y = -\gamma_y + 6\alpha x + 2\alpha y, \\
\sigma_{xy} &= -\frac{\partial^2 A}{\partial x \partial y} \Rightarrow \sigma_{xy} = -2\alpha x - 2\alpha y.
\end{align*}
\]

The boundary conditions on the OA wall of the dam, are written:

The traction vector being on the wall OB, since the atmospheric is neglected, the boundary conditions are written:

\[
\vec{T}(M, \vec{n}_{ext}) = \vec{\sigma} \cdot (-\vec{e}_y) = \gamma_e x \vec{e}_y
\]

Which leads to:

\[
\begin{align*}
\sigma_{xy}(x, 0) &= 0, \\
\sigma_y(x, 0) &= -\gamma_e x.
\end{align*}
\]

From where:

\[2b = 0 \quad \text{and} \quad 6d = \gamma_y - \gamma_e \]

The stress tensor components finally have for expressions:

\[
\begin{align*}
\sigma_x &= \left(\frac{\gamma_e}{\tan^2 \alpha} - \gamma_y\right)x + \left(\frac{\gamma_y - \gamma_e}{\tan^2 \alpha}\right)y, \\
\sigma_y &= -\gamma_y x, \\
\sigma_{xy} &= -\frac{\gamma_e}{\tan^2 \alpha} y.
\end{align*}
\]